

NPTEL ONLINE CERTIFICATION COURSES

Financial Institutions and Markets

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Bond Analysis-I

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Basic Features of a Bond

- Pay a fixed amount of interest periodically to the holder of record

- Repay a fixed amount of principal at the date of maturity

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Bond Characteristics

- Intrinsic features
- Types of issues
- Indenture provisions

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Intrinsic Features

- Coupon
  - Indicates the income that the bond investor will receive over the life of the issue.
- Term to Maturity
  - The date or the number of years before a bond matures.
    - Term Bond: Single Maturity Date
    - Serial Obligation Bond: Series of Maturity Dates
- Principal or Par value
  - It is the original value of obligation
- Types of Ownership
  - Bearer Bond
  - Registered Bond

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Types of Issues

- Secured (Senior) Bond
  - Backed by a legal claim on some specified property of the issuer in the case of default.
- Unsecured Bond (Debentures)
  - Backed by the promise of the issuer to pay interest and principal on a timely basis.

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Indenture Provisions

- It is the contract between the issuer and the bond holders specifying the issuer's legal requirements.
- A trustee acting on behalf of the bond holders ensures that all the indenture provisions are met including the timely payment of interest and principal.

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Price Quotes

- Many traders quote bond prices as a percentage of their par value.
- For example, if a bond is selling at par, it would be quoted at 100 (100% of par).
- A bond with a face value of Rs.10,000 and quoted at 80-1/8 would be selling at (.80125)(Rs.10,000) = Rs. 8,012.50

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Price Quotes

- When a bond's price is quoted as a percentage of its par, the quote is usually expressed in points and fractions of a point, with each point equal to Rs.1.

- Thus, a quote of 97 points means that the bond is selling for Rs. 97 for each Rs. 100 of par.

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Fractions

- Fractions can be in thirds, eighths, quarters, halves, or 64ths.
- On a $. 100 basis, a 1/2 point is $0.50 and a 1/32 point is $. 0.03125.
- A price quote of 97-4/32 (97-4) is 97.125 for a bond with a 100 face value.
- Bonds expressed in 64ths are sometime denoted in the financial pages with a plus sign (+); for example, 100.2+ would indicate a price of 100 2/64.

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Basis Points

- Fractions on yields are often quoted in terms of basis points (bp).
- A bp is equal to 1/100 of a percentage point.
  - 6.5% may be quoted as 6% plus 50 bp or 650 bp
  - An increase in yield from 6.5% to 6.55% would represent an increase of 5 bp

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Bid and Ask Price / Yield

- The ***bid price*** is the price the dealer is willing to pay for the bond.
- The ***ask price*** is the price the dealer is willing to sell the bond.
- The ***bid yield*** is the return expressed as a percent of the par value that the dealer wants if she buys the bill; this yield is often annualized.
- The ***ask yield*** is the rate that the dealer is offering to sell bills.
- For Treasury bills and some other securities, bid and ask yields are quoted as a ***discount yield***.

$$\text{Annual Discount Yield} = R_D = \frac{F - P_0}{F} \frac{360}{\text{Days to Maturity}}$$

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Bond Value

The value of a bond is the present value of its future cash flow (CF):

$$V_0^B = \sum_{t=1}^{M} \frac{CF_t}{(1+R)^t} = \frac{CF_1}{(1+R)^1} + \frac{CF_2}{(1+R)^2} + \cdots + \frac{CF_M}{(1+R)^M}$$

where:

CF_t = cash flow at t; principal and/or coupon

R = required return

M = term to maturity

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Bond Value

Assume the bond makes fixed coupon payments each year and principal at maturity.

$$V_0^B = \sum_{t=1}^{M} \frac{C}{(1+R)^t} + \frac{F}{(1+R)^M}$$

where :

C = annual coupon

F = principal

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Bond Value

$$V_0^B = \sum_{t=1}^{M} \frac{C}{(1+R)^t} + \frac{F}{(1+R)^M}$$

$$V_0^B = C\sum_{t=1}^{M} \frac{1}{(1+R)^t} + \frac{F}{(1+R)^M}$$

$$V_0^B = C[PVIF_a(R, M)] + \frac{F}{(1+R)^M}$$

$$V_0^B = C\left[\frac{1-1/(1+R)^M}{R}\right] + \frac{F}{(1+R)^M}$$

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Example

10-year, 9% annual coupon bond (9% of par), with F = $1,000 and required return of 10%.
What is the value of the bond?

$$V_0^B = \sum_{t=1}^{M} \frac{C}{(1+R)^t} + \frac{F}{(1+R)^M}$$

$$V_0^B = \sum_{t=1}^{10} \frac{\$90}{(1.10)^t} + \frac{\$1000}{(1.10)^{10}}$$

$$V_0^B = C\left[\frac{1-1/(1+R)^M}{R}\right] + \frac{F}{(1+R)^M}$$

$$V_0^B = \$90\left[\frac{1-1/(1.10)^{10}}{.10}\right] + \frac{\$1000}{(1.10)^{10}} = \$938.55$$

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Semi-Annual Coupon Payments

- The convention is to quote the required rate as a simple annual interest rate.
- When cash flows are semiannual, the convention is to use one-half the simple annual rate as the periodic interest rate to discount the cash flows.

$$V_0 = \sum_{t=1}^{N} \frac{C}{(1+R)^t} + \frac{F}{(1+R)^N} = \frac{C}{(1+R)^1} + \frac{C}{(1+R)^2} + \frac{C}{(1+R)^3} + \cdots + \frac{C}{(1+R)^N} + \frac{F}{(1+R)^N}$$

N = number of periods (= (Number of years)(2))

C = Semiannual coupon

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Semi-Annual Coupon Payments

10-year, 9% coupon bond, with F=$1,000, required return of 10%, and coupon payments made semiannually.

$$V_0^B = \sum_{t=1}^{2M} \frac{C^A/2}{(1+(R^A/2))^t} + \frac{F}{(1+(R^A/2))^{2M}}$$

$$V_0^B = \sum_{t=1}^{20} \frac{\$45}{(1.05)^t} + \frac{\$1000}{(1.05)^{20}}$$

$$V_0^B = C^A/2\left[\frac{1-1/(1+(R^A/2))^{2M}}{R^A/2}\right] + \frac{F}{(1+(R^A/2))^{2M}}$$

$$V_0^B = \$45\left[\frac{1-1/(1.05)^{20}}{.05}\right] + \frac{\$1000}{(1.05)^{20}} = \$937.69$$

Note: M = maturity in years

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n-Coupon Payments Per Year

- The rule for valuing semi-annual bonds is easily extended to valuing bonds paying interest even more frequently.

- For example, to determine the value of a bond paying interest four times a year, we would quadruple the number of annual periods and quarter the annual coupon payment and discount rate.

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n-Coupon Payments Per Year

In general, if we let n be equal to the number of payments per year (i.e., the compounding per year), M be equal to the maturity in years, N = number of periods to maturity = nM, and, as before, R^A be the discount rate quoted on an annual basis, then we can express the general formula for valuing a bond as follows:

$$V_0^B = \sum_{t=1}^{nM} \frac{C^A / n}{(1+(R^A/n))^t} + \frac{F}{(1+(R^A/n))^{nM}}$$

$$V_0^B = C^A / n \left[\frac{1 - 1/(1+(R^A/n))^{nM}}{R^A/n} \right] + \frac{F}{(1+(R^A/n))^{nM}}$$

Note : M = maturity in years

n = number of payments per year

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- Reilly, F. K., and K. C. Brown. *Investment analysis and portfolio management*, 10e. Cengage Learning, 2012.
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Bond Analysis-II

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Compounding Frequency

- The 10% annual rate is the rate with one annualized compounding. With one annualized compounding, we earn 10% every year and $100 would grow to equal $110 after one years:

$$\$100(1.10) = \$110$$

- If the simple annual rate were expressed with semi-annual compounding, then we would earn 5% every six months with the interest being reinvested; in this case, $100 would grow to equal $110.25 after one year:

$$\$100(1.05)^2 = \$110.25$$

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Compounding Frequency Cont…

- If the rate were expressed with monthly compounding, then we would earn 0.8333% (10%/12) every month with the interest being reinvested; in this case, $100 would grow to equal $110.47 after one year:

$$\$100(1.008333)^{12} = \$110.47$$

- If we extend the compounding frequency to daily, then we would earn 0.0274% (10%/365) daily, and with the reinvestment of interest, a $100 investment would grow to equal $110.52 after one year:

$$\$100(1+(0.10/365))^{365} = \$110.52$$

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Effective Rate

The rate that includes the reinvestment of interest (or compounding) is known as the ***effective rate***.

Effective Rate = $(1+(R^A/n))^n - 1$

Where, R^A= Simple annual rate

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Continuous Compounding

- When the compounding becomes large, then we approach towards *continuous compounding*.

- For cases in which there is continuous compounding, the future value (FV) for an investment of A dollars M-years from now becomes:

$$FV = A\,e^{RM}$$

  - where e is the natural exponent (equal to the irrational number 2.71828).

- Thus, if the 10% simple rate were expressed with continuous compounding, then $100 would grow to equal $110.52 after one year:

$$\$100e^{(.10)(1)} = \$110.52$$

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Continuous Compounding

- The present value of a future receipt (FV) with continuous compounding is

$$A = PV = \frac{FV}{e^{RM}} = FVe^{-RM}$$

- If R = 0.10, a bond paying $100 two years from now would currently be worth $81.87, given continuous compounding:

$$PV = \$100\, e^{-(0.10)(2)} = \$81.87$$

- Similarly, a bond paying $100 each year for two years would be currently worth $172.36:

$$PV = \sum_{t=1}^{2} \$100e^{-(.10)(t)} = \$100e^{-(.10)(1)} + \$100e^{-(.10)(2)} = \$172.36$$

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Continuous Compounding

If we assume continuous compounding and a discount rate of 10%, then the value of a 10-year, 9% bond would be:

$$V_0^b = \sum_{t=1}^{M} C^A e^{-Rt} + F e^{-RM}$$

$$V_0^b = \sum_{t=1}^{10} \$90 e^{-(.10)(t)} + \$1000 e^{-(.10)(10)} = \$908.82$$

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Valuation of Zero-Coupon Bond

- This types of bonds do not make any periodic coupon payments.
- The investor realizes interest as the difference between the maturity value and the purchase price.
- These bonds are called zero-discount bonds, zero coupon bonds (also called pure discount bonds (PDB)).

$$V_0^B = \frac{F}{(1+R)^M}$$

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Example

A zero-coupon bond maturing in 10 years and paying a maturing value of $1,000, the required rate is 10%, the value of the bond:

$$V_0^B = \frac{1{,}000}{(1.10)^{10}} = \$385.54$$

If the convention is to double the number of years and half the annual discount rate i.e. a semi-annual rate of 5%, effective annual rate of 10.25% (= $1.05)^2 - 1$), the value of the bond:

$$V_0^B = \frac{1{,}000}{(1.05)^{20}} = \$376.89$$

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Valuation of Zero-Coupon Bond with Maturity of Less than One Year

Let on March 1 a zero coupon bond promising to pay $1000 on September 1 (184 days) and trading at an annual rate of 8%, the value will be:

$$V = \$1000 / (1.08)^{(184/365)} = \$96.19$$

- The choice of time measurement used in valuing bonds is known as the ***day count convention***.
- The day count convention is defined as the way in which the ratio of the number of days to maturity (or days between dates) to the number of days in the reference period (e.g., year) is calculated.
  - A day count convention of actual days to maturity to actual days in the year (actual/actual)
  - A day count convention of 30-day months to maturity to a 360 days in the year (30/360)

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Bond Price Relation 1

Relation between coupon rate, required rate (discount rate), bond value (price), and face value (principal):

$$\text{Let } C^R = \text{coupon rate} = C/F$$

$$\text{If } C^R < R \Rightarrow V < F \Rightarrow \text{discount bond}$$

$$\text{If } C^R = R \Rightarrow V = F \Rightarrow \text{par bond}$$

$$\text{If } C^R > R \Rightarrow V > F \Rightarrow \text{premium bond}$$

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Bond Price Relation 1 Cont…

- When yields in the marketplace rise above the coupon rate, the price of the bond adjust so that investors contemplating the purchase can realize some additional return.
- When the yield in the market is below the coupon, the bond must sell above its par value

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Bond Price Relation 1 Cont…

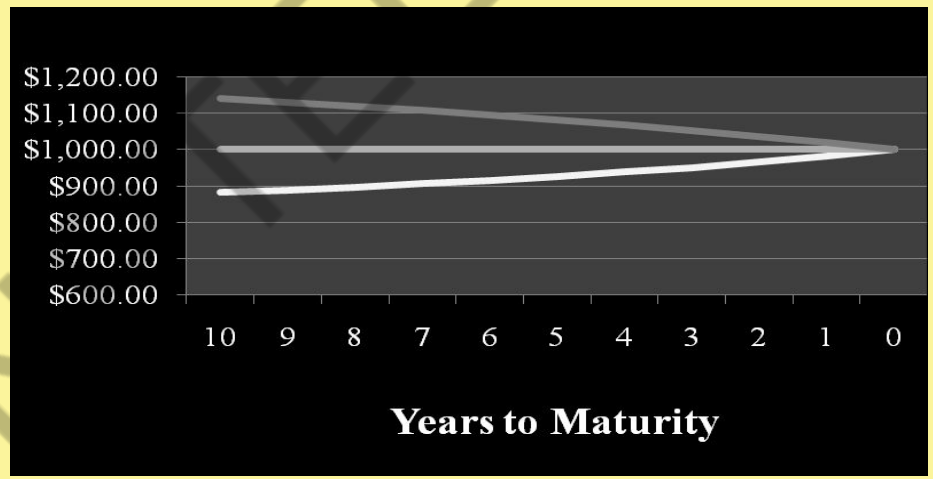
- If the require rate does not change, what will happen to the value of the bond from the time the bond is purchased to maturity?
  - For a bond selling at par, as the bond move closer to maturity, the bond will continue to sell at par; i.e., the price of the bond will remain constant as the bond moves toward maturity.
  - The bond price will not remain constant if it is selling at a premium or discount.
    - A discount bond increases in price as it moves towards maturity (assuming the required rate does not change).
    - A premium bond decreases in price as it moves towards maturity (assuming the required rate does not change).

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Example

Par value: $1000, Coupon= 9 %, Maturity: 10 Years

Year	Discount Bond Price of Bond Selling to Yield 11%	Par Bond Price of Bond Selling to Yield 9%	Premium Bond Price of Bond Selling to Yield 7%
10	$882.22	$1,000.00	$1,140.47
9	$889.26	$1,000.00	$1,130.30
8	$897.08	$1,000.00	$1,119.43
7	$905.76	$1,000.00	$1,107.49
6	$915.39	$1,000.00	$1,095.33
5	$926.08	$1,000.00	$1,082.00
4	$937.95	$1,000.00	$1,067.74
3	$951.13	$1,000.00	$1,052.49
2	$965.75	$1,000.00	$1,036.16
1	$981.98	$1,000.00	$1,018.69
0	$1,000.00	$1,000.00	$1,000.00



Bond Price Relation 2

Inverse relation between bond price (value) and rate of return.

$$\text{If } R\uparrow \Rightarrow V\downarrow$$

$$\text{If } R\downarrow \Rightarrow V\uparrow$$

$$\frac{\Delta V}{\Delta R} < 0$$

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Price-Yield Curve

It depicts the inverse relation between V and R. The Price-Yield curve for the 10-year, 9% coupon bond

Rate	Bond Value
7.00%	$1,140.47
7.50%	$1,102.96
8.00%	$1,067.10
8.50%	$1,032.81
9.00%	$1,000.00
9.50%	$968.61
10.00%	$938.55
10.50%	$909.78
11.00%	$882.22
11.50%	$855.81
12.00%	$830.49

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Bond Price Relation 3

The greater a bond's maturity, the greater its price sensitivity to interest rate changes. Symbolically:

$$\text{Let} \quad \varepsilon = \left| \frac{\%\Delta V}{\%\Delta R} \right|$$

$$\text{Greater} \quad M \quad \Rightarrow \quad \text{Greater} \quad \varepsilon$$

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Bond Price Relation 4

The smaller a bond's coupon rate, the greater its price sensitivity to interest rate changes. Symbolically:

$$\text{Let } \varepsilon = \left| \frac{\%\Delta V}{\%\Delta R} \right|$$

$$\text{Lower } C^R \Rightarrow \text{Greater } \varepsilon$$

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Johnson, R.S. (2010), Bond Valuation, Selection, and Management, Second Edition, John Wiley & Sons, Inc., Hoboken, New Jersy.

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Bond Analysis-III

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Current Yield and Nominal Yield

- ***Current yield*** of a bond is the ratio of its annual coupon to its closing price.
- ***Coupon rate***, C^R, is the contractual rate the issuer agrees to pay each period. It is expressed as a proportion of the annual coupon payment to the bond's face value:

$$C^R = \frac{\text{Annual Coupon}}{F}$$

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Yield to Maturity

- YTM is the rate that equates the price of the bond, P_0^B, to the PV of the bond's cash flow (CF); it is similar to the internal rate of return, IRR.
- In general, the yield on any investment is the interest rate that will make the present value of the cash flow from the investment equal to the price (or cost) of the investment.
- In our first example, if the price of the 10-year, 9% *annual* coupon bond were priced at $938.55, then we will get its YTM by solving the following equation:

$$P_0^B = \sum_{t=1}^{M} \frac{C}{(1+YTM)^t} + \frac{F}{(1+YTM)^M}$$

$$\$938.55 = \sum_{t=1}^{10} \frac{\$90}{(1+YTM)^t} + \frac{\$1000}{(1+YTM)^{10}} \Rightarrow YTM = .10$$

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Example

If the price of the 10-year, 9% coupon bond with *semi-annual payments with the par value $1000* were priced at $937.69, then its yield will be:

$$P_0^B = \sum_{t=1}^{N} \frac{C^A/2}{(1+YTM)^t} + \frac{F}{(1+YTM)^N}$$

$$\$937.69 = \sum_{t=1}^{20} \frac{\$45}{(1+YTM)^t} + \frac{\$1000}{(1+YTM)^{20}} \Rightarrow YTM = .05$$

$$\text{Simple Annual Rate} = (2)(.05) = .10$$

$$\text{Effective Annual Rate} = (1.05)^2 - 1 = .1025$$

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Yield to Maturity

- The YTM is the effective rate of return. As a rate measure, it includes:

1. Return from coupons
2. Capital gains or losses
3. Reinvestment of coupons at the calculated YTM

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Bond Equivalent Yield

- The rate on bonds are often quoted as a simple annual rate (with no compounding). For bonds with semi-annual coupon payments, this rate can be found by solving for the YTM on a bond using 6-month CFs and then multiplying that rate by 2. This rate is also known as the ***bond-equivalent yield***.
- Thus, the 10-year, 9% bond with semi-annual payments and trading at 937.69 would have a YTM for a 6-month period of 5% and a bond-equivalent yield of 10%. The effective rate is 10.25%.
- Bonds with different payment frequencies often have their rates expressed in terms of their bond-equivalent yield so that their rates can be compared to each other on a common basis.

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Average Rate to Maturity

- Unless the CFs are constant, there is no algebraic solution to finding the YTM. The YTM is found through an iterative process (trial and error).
- The YTM can be estimated using the ARTM (also referred to as the yield approximation formula):

$$\text{ARTM} = \frac{C + [(F - P_o^B)/M]}{(F + P_0^b)/2}$$

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Example

- The ARTM for the 9%, 10-year bond with annual payments trading at $938.55 is:

$$\text{ARTM} = \frac{\$90 + [(\$1000 - \$938.55)/10]}{(\$1000 + \$938.55)/2} = .0992$$

- The semi-annual ARTM for the 9%, 10-year bond with semi-annual payments and trading at $937.69 is:

$$\text{ARTM} = \frac{\$45 + [(\$1000 - \$937.69)/20]}{(\$1000 + \$937.69)/2} = .049663$$

$$\text{Annualized ARTM} = (2)(.049663) = .099325$$

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YTM on Zero-Coupon Bond

Algebraic solution to the YTM on zero-coupon bond (or pure discount bond (PDB)):

$$P_0^B = \frac{F}{(1+YTM)^M}$$

$$(1+YTM)^M = \frac{F}{P_0^B}$$

$$YTM = \sqrt[M]{\frac{F}{P_0^B}} - 1$$

Or

$$YTM = \left[\frac{F}{P_0^B}\right]^{1/M} - 1$$

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Example

Example: $P_0^B = \$800,\ F = \$1000,\ M = 3$ years

$$\text{YTM} = \left[\frac{\$1000}{\$800}\right]^{1/3} - 1 = .0772$$

Example: $P_0^B = \$800,\ F = \$1{,}000,\ M = 3$ years,

$n = \text{compound frequency} = 2$

$N = (2)(3) = 6$

$$\text{Semi} - \text{Annual YTM} = \left[\frac{\$1000}{\$800}\right]^{1/6} - 1 = .03789$$

$$\text{Bond} - \text{Equivalent Yield} = (2)(.03789) = .075782$$

Example: $P_0^B = \$96,\ F = \$100,\ \text{Maturity} = 182$ days,

$\text{Day} - \text{Count Convention} = \text{Actual}/365,$

$M = 182/365$

$$\text{YTM} = \left[\frac{\$100}{\$96}\right]^{365/182} - 1 = .0853$$

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Example

Algebraic solution to the YTM on a zero-coupon bond with continuous compounding:

$$P_0^B e^{Rt} = F$$

$$e^{Rt} = \frac{F}{P_0^b}$$

$$\ln(e^{Rt}) = \ln\left[\frac{F}{P_0^b}\right]$$

$$Rt = \ln\left[\frac{F}{P_0^b}\right]$$

$$R = \frac{\ln[F/P_0^b]}{t}$$

$$\text{Example: } P_0^B = \$96,\ F = \$100,\ \text{Maturity} = 182 \text{ days},$$

$$\text{Day Count Convention} = \text{Actual}/365,$$

$$M = 182/365$$

$$\text{YTM} = \frac{\ln[100/96]}{182/365} = .081868$$

Logarithmic Return: The rate of return expressed as the natural log of the ratio of its end-of-the-period value to it current value

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Yield to Call

- Many bonds have a call feature that allows the issuer to buy back the bond at a specific price known as the call price, CP.
- When the bond may be called and at what price are specified in the indenture.
- For some issues, the call price is the same.
- For other callable bonds, the call price depends on when the bond is called. In this case, there is a call schedule that specifies the call price for each call dates.

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Yield to Call

- Given a bond with a call option, the ***yield to call***, YTC, is the rate obtained by assuming the bond is called on the identified call date, CD.

- Like the YTM, the YTC is found by solving for the rate that equates the present value of the CFs to the market price.

$$P_0^B = \sum_{t=1}^{N} \frac{CF_t}{(1+YTC)^t} + \frac{CP}{(1+YTC)^N}$$

  - Where: N = number of periods to the identified call date

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Example

- Find the yield to call of a 10-year, 9% coupon bond, first callable in 5 years at a call price of $1100, paying interest semiannually and trading at $937.69?

$$\$937.69 = \sum_{t=1}^{10} \frac{\$45}{(1+\text{YTC})^t} + \frac{\$1100}{(1+\text{YTC})^{10}} \quad \Rightarrow \text{YTC} = .0610575$$

$$\text{Simple Annualized YTC} = (2)(.0610575) = .122115$$

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Yield to Put

- An issue can be putable, allowing the bondholder the right to sell the bond back to the issuer at a specified price – put price, PP.

- As with callable bonds, putable bonds can have a constant put price or a put schedule.

- When a bond is putable, the convention is to calculate the yield to put, YTP.

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Yield to Put

- YTP is rate that equates the present value of the bond's CF to the assumed put date plus the present value PP at the put date equal to the bond's price.
- That is, like the YTM and YTC, the YTP is found by solving for the rate that equates the present value of the CF to the market price.

$$P_0^B = \sum_{t=1}^{N} \frac{CF_t}{(1+YTP)^t} + \frac{PP}{(1+YTP)^N}$$

Where: N = number of periods to the identified put date

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Example

- Find our yield to put of a 10-year, 9% coupon bond, first putable in 5 years at a put price of $950, paying interest semiannually and trading at $937.69.

$$\$937.69 = \sum_{t=1}^{10} \frac{\$45}{(1+\text{YTP})^t} + \frac{\$950}{(1+\text{YTP})^{10}} \Rightarrow \text{YTP} = .04903870$$

$$\text{Simple Annualized YTP} = (2)(.04903870) = .09807741$$

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Yield to Worst

- The convention is for an investor to calculate the YTM , the yields to all possible call dates, and the yields to all possible put dates.

- The minimum of all the yields is called the ***yield to worst***.

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Bond Portfolio Yield

- The yield for a portfolio of bonds is found by solving the rate that will make the present value of the portfolio's cash flow equal to the market value of the portfolio.

- For example, a portfolio consisting of a two-year, 5% annual coupon bond priced at par (100) and a three-year, 10% annual coupon bond priced at 107.87 to yield 7% (YTM) would generate a three-year cash flow of $15, $115, and $110 and would have a portfolio market value of $207.87. Find out yield of the portfolio?

$$\$207.87 = \frac{\$15}{(1+y)^1} + \frac{\$115}{(1+y)^2} \frac{\$110}{(1+y)^3} \Rightarrow y = .062$$

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Reference

Johnson, R.S. (2010), Bond Valuation, Selection, and Management, Second Edition, John Wiley & Sons, Inc., Hoboken, New Jersy.

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Financial Institutions and Markets

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Bond Analysis-IV

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Total Return

- The total return (also call the realized yield) is a measure of the yield obtained by assuming the cash flows are to be reinvested to the investor's horizon (HD) at an assumed reinvestment rate and at the horizon the bond is sold at an assumed rate given the horizon is not at maturity.
- The total return is determined by
  - Estimating the horizon value, total monetary return and bond price at the horizon
  - Given the current price or value and the horizon value, solving for the rate (similar to the way one solve for the rate on a zero-coupon bond)

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Example

- Suppose an investor buys a 4-year, 10% coupon bond, paying coupons annually, and selling at its par value of $1,000. Assume the investor needs cash at the end of year 3 (HD = 3), is certain he can reinvest the coupons during the period in securities yielding 10%, and expects to sell the bond at his HD at a rate of 10%.
- To determine the investor's TR, we first need to find the HD value. This value is equal to the price the investor obtains from selling the bond at HD and the value of the coupons at the HD.

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Example Cont…

- In this case, the investor, at his HD, will be able to sell a one-year bond paying a $100 coupon and a $1,000 par at maturity for $1,000, given the assumed discount rate of 10%

$$P_0^b = \frac{\$100 + \$1{,}000}{(1.10)^1} = \$1{,}000$$

- The $100 coupon paid at the end of the first year will be worth $121, given the assumption it can be reinvested at 10% for two years and there is annual compounding, $100(1.10)^2 = $121
- The $100 received at the end of year two will, in turn, be worth $110 in cash at the HD, $100(1.10) = $110.
- The investor would receive his third coupon of $100.
- Combined, the investor would have $1,331 in cash at the HD: HD value = $1,331.
- The horizon value of $1,330 consists of a bond valued at $1,000, coupons of $300, and interest earned from reinvesting coupons of $31 (HD coupon value – total coupon received = $331 – $300 – $31).

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Example Cont...

$$\text{Coupon Value at HD} = \sum_{t=0}^{HD-1} C(1+R)^t$$

$$\text{Coupon Value at HD} = C\sum_{t=0}^{HD-1} (1+R)^t$$

$$\text{Coupon Value at HD} = C\ FVIF_a$$

$$\text{Coupon Value at HD} = C\left[\frac{(1+R)^{HD}-1}{R}\right]$$

$$\text{Coupon Value at HD} = \$100\left[\frac{(1.10)^3-1}{.10}\right] = \$331$$

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Example Cont…

- Given the HD value of $1,331, the TR is found in the same way as the YTM for a zero-coupon bond.

$$P_0^b = \frac{\text{HD Value}}{(1+\text{TR})^{\text{HD}}}$$

$$(1+\text{TR})^{\text{HD}} = \frac{\text{HD Value}}{P_0^b}$$

$$\text{TR} = \left[\frac{\text{HD Value}}{P_0^b}\right]^{1/\text{HD}} - 1$$

$$\text{TR} = \left[\frac{\$1{,}331}{\$1{,}000}\right]^{1/3} - 1 = .10$$

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Duration

- A bond's ***duration*** (D) can be defined as the weighted average of the bond's time periods, with the weights being each time period's relative present value of its cash flow
- It is the weighted average on a present value basis of the time to full recovery of the principal and interest payments on a bond. It measures the weighted average maturity of a bond’s cash flows on a present value basis.

$$D = \sum_{t=1}^{N} t \frac{PV(CF_t)}{P_0^b}$$

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Example

- Find out the duration of a 4-year, 9% annual coupon bond with par value $1000 given a flat yield curve at 10%

t	CF_t	$CF_t/(1.10)^t$	$PV(CF_t)/P^B$	$t[PV(CF_t)/P^B]$
1	90	81.818	.084496	0.084496
2	90	74.380	.076815	0.153630
3	90	67.618	.069832	0.209496
4	1090	744.485	.768857	3.075428
		$P^B = 968.30$		D = 3.52

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Duration of a Bond Portfolio

- The duration of a bond portfolio, D_p, is simply the weighted average of each of the bond's durations (D_i), with the weights being the proportion of investment funds allocated to each bond (w_i):

$$D_p = \sum_{i=1} w_i D_i$$

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Duration as a Price Sensitivity Measure

- Though duration is defined as the weighted average of a bond's time periods, it is also an important measure of volatility.
- As a measure of volatility, duration is defined as the percentage change in a bond's price ($\%\Delta P = \Delta P/P_0$) given a small change in yield, dy.
- Mathematically, duration is obtained by taking the derivative of the equation for the price of a bond with respect to the yield, then dividing by the bond's price and expressing the resulting equation in absolute value.

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Duration as a Price Sensitivity Measure Cont…

$$\text{Duration} = \frac{dP/P}{dy} = \frac{1}{(1+y)}\left(\sum_{t=1}^{N} t\frac{PV(CF_t)}{P_0^B}\right)$$

- dP/P_0 = percentage change in the bond's price
- dy = small change in yield
- N = number of periods to maturity (M)

- The bracketed expression is the weighted average of the time periods, defined in the last section as duration.
- Formally, the weighted average of the time periods is called ***Macaulay's duration***, and the equation, which defines the percentage change in the bond's price for a small change in yield in absolute value, is called the ***modified duration***.

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Duration as a Price Sensitivity Measure Cont…

- Note that the price of a bond that pays coupons each period and its principal at maturity is

$$P_0^B = C\left[\frac{1-(1/(1+y))^N}{y}\right] + \frac{F}{(1+y)^N}$$

- Taking the first derivative of this equation, dividing through by P, and expressing the resulting equation in absolute value provides a measure of duration for a bond paying principal at maturity:

$$\text{Modified Duration} = \frac{\frac{C}{y^2}\left[1 - \frac{1}{(1+y)^N}\right] + \frac{N[F - (C/y)]}{(1+y)^{N+1}}}{P_0^b}$$

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Duration as a Price Sensitivity Measure Cont…

- The above measures of duration are defined in terms of the length of the period between payments.
- Thus, if the cash flow is distributed annually, duration reflects years; if cash flow is semi-annual, then duration reflects half years.
- The convention is to express duration as an annual measure.
- Annualized duration is obtained by dividing duration by the number of payments per year (n):

$$\text{Annualized Duration} = \frac{\text{Duration for bond with n – payments per year}}{n}$$

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Properties of Duration

- The lower the coupon rate, the greater the duration.
- The longer the terms to maturity, the greater the duration.
- For zero-coupon bonds, Macaulay's duration is equal to the bond's term to maturity (N) and the modified duration is equal N/(1+y).
- The higher the yield to maturity, the lower the duration.

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Convexity

- The price-yield curve is not linear, but convex from below (bowed-shaped.
- Convexity measures how bowed-shaped the price-yield curve is.
- ***Convexity*** is a measure of the change in the slope of the price-yield curve – *second-order derivative*.

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Convexity Cont…

- Convexity means that the slope of the price-yield curve (Δ = dP/dy) gets smaller as you move down the curve or as the YTM increases. Mathematically, convexity is the change in the slope of the price-yield curve for a small change in yield; it is the second-order derivative.
- It is derived by taking the derivative of equation for dP/dy and dividing the resulting equation by the current price.

$$\text{Convexity} = \frac{1}{P_0^b}\left[\sum_{t=1}^{N}\frac{t(t+1)(CF_t)}{(1+y)^{t+2}}\right]$$

- The convexity of a bond that pays a fixed coupon each period and the principal at maturity is

$$\text{Convexity} = \frac{\frac{2C}{y^3}\left[1-\frac{1}{(1+y)^N}\right]-\frac{2CN}{y^2(1+y)^{N+1}}+\frac{N(N+1)[F-(C/y)]}{(1+y)^{N+2}}}{P_0^b}$$

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Convexity Cont…

- Like duration, convexity reflects the length of periods between cash flows.

- The annualized convexity is found by dividing convexity, measured in terms of n-payments per year, by n^2:

$$\text{Annualized Convexity} = \frac{\text{Convexity for bond with n – payments per year}}{n^2}$$

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Example

- Find the convexity for the 10-year, 9% coupon bond with semi-annual payments with bond value 100.

$$\text{Convexity } (1/2 \text{ yrs}) = \frac{\frac{2(4.5)}{.045^3}\left[1-\frac{1}{(1.045)^{20}}\right] - \frac{2(4.5)(20)}{(.045)^2(1.045)^{21}} + \frac{(20)(21)[100-(4.5/.045)]}{(1.045)^{22}}}{100} = 225.43$$

$$\text{Annualized Convexity} = \frac{225.43}{2^2} = 56.36$$

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Properties of Convexity

- As the yield increases (decreases), the convexity of the bond decreases (increases).
- For a given yield and maturity, the lower the coupon, the greater the convexity.
- For a given yield and modified duration, the lower the coupon, the smaller the convexity.

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Reference

Johnson, R.S. (2010), Bond Valuation, Selection, and Management, Second Edition, John Wiley & Sons, Inc., Hoboken, New Jersy.

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Bond Markets in India

Bond Markets in India

- Government dated securities market
- Corporate bond market
- Public sector unit bonds

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Government Security Market

- Dated Government securities are longer term securities and carry a fixed or floating coupon (interest rate) paid on the face value, payable at fixed time periods (usually half-yearly).
- The tenor of dated securities can be up to 30 years.
- The Public Debt Office (PDO) of the RBI acts as the registry / depository of Government securities and deals with the issue, interest payment and repayment of principal at maturity.

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Importance of Government Securities Market

- It sets a benchmark for the rest of the market.
- Acts as the benchmark for pricing corporate securities of varying maturities.
- Essential for the conduct and effectiveness of Open Market Operations (OMOs)
- Free of default risk or credit risk
- GSM facilitates public borrowings at reasonable cost and the avoidance of automatic monetisation of government deficit
- GSM provides flexibility to the authorities in their task of debt management
- Plays a crucial role in the monetary policy transmission mechanism

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Existing Structure and Organization of Dated GSM in India

- Major players in the Government securities market include reserve bank of India, commercial banks and primary dealers besides institutional investors like insurance companies
- Foreign Institutional Investors (FIIs) are allowed to participate in the Government securities market within the quantitative limits prescribed from time to time. Corporates also buy/ sell the government securities to manage their overall portfolio risk.

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Existing Structure and Organization of Dated GSM in India Cont…

- The investors in this market can be classified into different segments such as
- Wholesale market which comprises the players like banks, financial institutions, insurance companies, primary dealers and mutual funds
- Middle segment comprising corporate, provident funds, non banking financial companies and co-operative banks
- Retail segment consisting of individuals and non-institutional investors.

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Existing Structure and Organization of Dated GSM in India Cont…

- The holders can maintain their securities in dematerialsed form in either of the two ways:
- SGL Account: Reserve Bank of India offers Subsidiary General Ledger Account (SGL) facility to select entities who can maintain their securities in SGL accounts maintained with the Public Debt Offices, of the Reserve Bank of India.
- Gilt Account: As the eligibility to open and maintain an SGL account with the RBI is restricted, an investor has the option of opening a Gilt Account with a bank or a Primary Dealer which is eligible to open a Constituents' Subsidiary General Ledger Account (CSGL) with the RBI.

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Primary Market Issuance of Dated Government Securities

Yield-based Auction of a New Security:

- Maturity Date: 6 August 2024
- Coupon: It is determined in the auction (8.22 percent as shown in the following illustration)
- Auction date: 5 August 2014
- Auction settlement date: 6 August 2014*
- Notified Amount: ₹1000 crore

*Settlement is done on 6 August, 2014, under $T+1$ cycle. If the settlement date is a holiday under $T+1$ cycle, then the next working will be considered as settlement date.

Details of bids received in the increasing order of bid yields

Bid No.	*Bid yield*	*Amount of bid (₹ crore)*	*Cummulative amount (₹ Crore)*	*Price* with coupon as 8.22%*
1	8.19%	300	300	100.19
2	8.20%	200	500	100.14
3	8.20%	250	750	100.13
4	8.21%	150	900	100.09
5	8.22%	100	1000	100.00
6	8.22%	100	1100	100.00
7	8.23%	150	1250	99.93
8	8.24%	100	1350	99.87

The issuer would get the notified amount by accepting bids up to 5. Since the bid number 6 also is at the same yield, bid numbers 5 and 6 would get allotment *pro-rata* so that the notified amount is not exceeded. In the above case each would get ₹50 crore. Bid numbers 7 and 8 are rejected as the yields are higher than the cut-off yield.

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Primary Market Issuance of Dated Government Securities

Price-based auction of an existing security 8.24% GS 2024

- Maturity Date: 22 August, 2024
- Coupon: 8.24%
- Auction date: 5 August, 2014
- Auction settlement date: 6 August, 2014*
- Notified Amount: ₹1000 crore

*Settlement is done on 6 August, 2014 under $T+1$ cycle. If the settlement date is a holiday under $T+1$ cycle, then the next working will be considered as settlement date.

Details of bids received in the decreasing order of bid price

Bid no.	*Price of bid*	*Amount of bid (₹ Crore)*	*Implicit yield*	*Cumulative amount*
1	100.31	300	8.1912%	300
2	100.26	200	8.1987%	500
3	100.25	250	8.2002%	750
4	100.21	150	8.2062%	900
5	100.20	100	8.2077%	1000
6	100.20	100	8.2077%	1100
7	100.16	150	8.2136%	1250
8	100.15	100	8.2151%	1350

The issuer would get the notified amount by accepting bids up to 5. Since the bid number 6 also is at the same yield, bid numbers 5 and 6 would get allotment in proportion so that the notified amount is not exceeded. In the above case each would get ₹50 crore. Bid numbers 7 and 8 are rejected as the price quoted is less than the cut-off price.

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Trading of Dated Government Securities in Secondary Market

- **Over the Counter (OTC)/ Telephone Market**
  - Transactions undertaken between market participants in the OTC/telephone market are expected to be reported on the NDS platform within 15 minutes after the deal is put through over telephone.
- **Negotiated Dealing System**
  - The Negotiated Dealing System (NDS) for electronic dealing and reporting of transactions in government securities was introduced in February 2002. It facilitates the members to submit electronically, bids or applications for primary issuance of Government Securities when auctions are conducted.
- **Stock Exchanges**
  - Introduced for retail investors through the stock exchanges like NSE and BSE

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Corporate Bond Market

- Corporate debt is a security issued by a corporation and sold to investors.
- The backing for the bond is usually the payment ability of the company, which is typically money to be earned from future operations. In some cases, the company's physical assets may be used as collateral for bonds.
- Corporate bonds are considered higher risk than government bonds. As a result, interest rates are almost always higher.

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Classification of Corporate Bonds in India

Based on issuer	*Based on maturity*	*Based on coupon*	*Based on option*	*Based on redemption*
Issued by corporates	**Short-term:** *Maturity* period less than 1 year	**Zero coupon bonds:** No coupons are paid. The bond is issued at a discount to its face value, at which it will be redeemed.	**Put option:** This feature gives bondholders the right but not the obligation to sell their bonds back to the issuer at a predetermined price and date. These bonds generally protect investors from interest rate risk.	**Single redemption:** In this case, principal amount of bond is paid at the time of maturity only.
Issued by banks	**Medium term:** Maturity period between 1 and 5 year.	**Fixed coupon:** Coupon rate that remains constant throughout the life of the bond.	**Call option:** This feature gives a bond issuer the right, but not the obligation, to redeem his issue of bonds before the bond's maturity at predetermined price and date.	**Multiple redemption/ Amortizing bond:** A bond, in which payment made by the borrower over the life of the bond, includes both interest and principal, is called an amortizing bond.
Issued by Public Sector Undertakings	**Long Term:** Maturity period more than 5 years	**Floating coupon:** Coupon rates are reset periodically based on benchmark rate		
Issued by Local Bodies	**Perpetual:** no Maturity			

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PSU Bonds

- PSU bonds are medium or long term debt instruments issued by Public Sector Undertakings (PSUs). The term usually denotes bonds issued by the central PSUs (i.e. PSUs funded by and under the administrative control of the Government of India).
- Most of the PSU Bonds are sold on Private Placement Basis to the targeted investors at Market Determined Interest Rates. Often investment bankers are roped in as arrangers to this issue.
- Most of the PSU Bonds are transferable and endorsement at delivery and are issued in the form of Usance Promissory Note. PSU Bonds are issued in demat form.
- In order to attract the investors and increase liquidity, issuers get their bonds rated by rating agencies like CRISIL, ICRA, CARE, etc. Some of the issues may be guaranteed by Central / State Government enabling them to get a better rating.

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PSU Bonds Cont…

- The maturity of PSU bonds varies between 5 to 10 years. The default rates of PSU bonds are negligible and they are perceived as quasi sovereign bonds.
- The bonds issued by PSUs are of two types, *viz.*, taxable and tax-free.
- The provisions of tax free bonds as contained in Section 10(15)(iv)(h) of IT Act allows complete exemption of interest income arising from notified PSUs bonds .
- The major investors in PSU bonds are banks, mutual funds, insurance companies, provident funds and individuals.

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- Bhole, L.M and Mahakud, J (2017), Financial Institutions and Markets, Sixth Edition, McGraw Hill Education (India) Private Limited, Chennai, India

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